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MEASURES OF PLANT AGGREGATION BASED ON CONTAGIOUS DISTRIBUTION

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INTRODUCTION

THE distribution of plant species in plant communities has been studied from several different points of view, but few mathematical analyses of the problem have been made. The labor in the complete mapping of all the individuals of a single species in a community usually precludes this type of presentation and less satisfactory schemes must be used.

The usual method of obtaining quadrat frequency data on a single species is the sampling of a representative area of the community by a number of scattered areas (quadrats) and the determination of the number of plants of that species in each quadrat. If the plants are distributed completely at random over the area and they have no influence on each other it can be shown statistically that the distribution will be of the Poisson type (see below). This was established for a number of species in certain grassland communities by Blackman (1935) and by Ashby (1935). The presence of clones, rhizomes, and runners and other clumping factors, however, vitiates the fundamental assumptions and under these conditions the data do not follow a simple Poisson law. Among the workers who have studied the general problem of nonrandomness are Rommel (1930), McGinnies (1934), Ashby (1935, 1948), Blackman (1935, 1942), Clapham (1936), Fracker and Brischle (1944), Cole (1946), Archibald (1948), Whitford (1949), Thomas (1949), and Curtis and McIntosh (1950). Many of these investigators proposed measures of nonrandomness (see below), but the general problem of obtaining a measure of the size of the clumps was not taken up by them.

Two similar lines of approach to these nonrandom distributions are afforded by the use of "contagious" distribution functions such as those of Neyman (1939) and Thomas (1949), where the mathematical model assumes that primary centers ("clusters") are distributed at random and that individual plants are then distributed, again at random, about these primary centers. These models have proved more or less successful in representing certain plant-distribution data (Archibald, 1948; Thomas, 1949). A typical example of such an excellent agreement is provided by these authors for the distribution of Armeria maritima in an English marsh (Table I). This material is noteworthy in that both of these models fit satisfactorily the increase in the observed number of quadrats at two plants per quadrat. As can be seen, the simple Poisson series can not possibly fit this particular type of distribution.

TABLE I

Distribution of Armeria maritima

Plants Per Quadrat	Number of Quadrats			
	Observed	Calculated		
		Poisson*	Neyman †	Thomas ‡
0	57	20.6	54.9	57.0
1	6	32.5	7.9	6.0
2	12	25.7	10.1	10.1
3	5	13.5	9.0	9.5
4	5	5.4	6.5	6.5
5	5	1.7	4.3	4.1
6	7	0.5	2.8	2.6
7	1	0.1	1.8	1.6
8	0	0.0	1.1	1.0
9	1	0.0	0.9	0.6
10	1	0.0	0.4	0.3
11 and over	0	0.0	0.3	0.7
Total	100	100.0	100.0	100.0

* Poisson distribution (Thomas, 1949).
† Neyman's Type A contagious distribution (Archibald, 1948).
‡ Thomas' double Poisson series, Method II (Thomas, 1949).

The number of "plants per cluster" is one of the parameters of both these mathematical models. For the Armeria data the results are: Neyman, 2.39; Thomas, 2.68. These quantities are of the same magnitude and can be logically related to a small degree of clumping of the plants. In other words, the mathematical model appears to possess some relation to reality for this example. For Plantago maritima (Archibald, 1948; Thomas, 1949) a similar excellent agreement was found between the calculated series and the quadrat frequency data. The number of "plants per cluster" was found to be 2.16 (Thomas), again a reasonable value. Thomas pointed out, however, that although her mathematical set-up was so designed that the parameters of the distribution might be capable of physical interpretation, sufficient data were not then available to test whether the distribution parameters do, in fact, measure the actual number of "plants per cluster" and "clusters per quadrat."

A plot of the actual position of every plant of a given species in an area usually reveals some type of clumping or clustering which is quite evident to the eye but rather difficult to establish rigorously since the cluster boundaries are seldom distinct and it is difficult to tell where one cluster ends and another starts. An excellent example is shown by the position plots of several species in an old-field community on the Edwin S. George Reserve of the University of Michigan, Livingston County, southeastern Michigan. Cain and Evans (1952) have mapped and plotted the distribution patterns of Solidago rigida, Liatris aspera and Lespedeza capitata, as they occurred within an area of approximately two acres in this community. The number of plants in all the 7640 one-meter-square quadrats believed to be a valid part of the community are studied below. Casual observation of the position plots suggests that Liatris is more definitely clumped than Solidago and that Lespedeza is still more clumped.

The object of this study was to determine whether the clusters of the contagious series have any relation to these major clumps which are evident to the eye. The results strongly suggest that the clusters of these series are more closely related to the minor clumps or groups of plants that are immediately adjacent rather than to the evident clumps (Cain and Evans, 1952), and that one must look elsewhere for a simple statistical measure of the degree of major clumping which is evident to the casual observer. Such a measure may be related to the average spacing parameter described by Dice (1952).

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CONTAGIOUS DISTRIBUTIONS APPLIED TO OLD-FIELD DATA

The observed frequency data for the three populations under consideration and the goodness of fit of the double Poisson series of Thomas to the values are shown in Tables II to IV. The values have been calculated by the two methods suggested by Thomas, the moment solution (her Method I) and the maximum likelihood solution (her Method II). By a mathematical coincidence the second method leads to an exact fit of the values for 0 and 1 plant per quadrat. Similar comparisons with Neyman's series are shown in Tables II and IV. The calculated values of the Neyman series in Table II are more strictly comparable to Method I, the moment solution of Thomas, since the constants of the Neyman series were also obtained by the method of moments. The application of the method of maximum likelihood to Neyman's Type A contagious distribution has been described by Shenton (1949). Unfortunately, the calculation is involved and requires the usual moment solution as an initial approximation. Table IV includes a comparison of the results by this method for the Lespedeza population.

The three species provide examples of an excellent fit (Solidago), a fair fit (Liatris), and a very bad fit (Lespedeza), but in all cases the size of the clusters of the Thomas model lies between 1 and 3 plants (average for the moment solutions, Method I: 1.6; average for the maximum likelihood solutions, Method II: 1.3). The size of the clusters of the Neyman model are 0.23 (Solidago) and 1.76 (Lespedeza) for the moment solutions. These values suggest that the Solidago distribution is

TABLE II

Distribution of Solidago rigida

Plants Per Quadrat	Number of Quadrats			
	Observed	Calculated		
		Thomas I	Thomas II	Neyman
0	7229	7230.5	7229.0	7229.7
1	356	351.9	356.0	354.2
2	49	52.2	50.0	50.1
3	5	5.0	4.6	5.4
4	1	0.4	0.4	0.5
5 or over . .	0	0.0	0.0	0.1
Total . . .	7640	7640.0	7640.0	7640.0
Plants per cluster	......	1.124	1.116	0.234
Clusters per quadrat	......	0.0551	0.0553	0.265
Plants per quadrat	......	0.0619	0.0617	0.0619

TABLE III

Distribution of Liatris aspera

Plants Per Quadrat	Number of Quadrats		
	Observed	Calculated	
		Thomas I	Thomas II
0	7403	7420.4	7403.0
1	183	140.0	183.0
2	34	62.3	46.7
3	14	14.4	6.5
4	4	2.4	0.7
5	1	0.4	0.1
6	1	0.1	0.0
7 or over	0	0.0	0.0
Total.	7640	7640.0	7640.0
Plants per cluster. .	......	1.436	1.243
Clusters per quadrat	......	0.0292	0.0315
Plants per quadrat. .	......	0.0419	0.0392

TABLE IV

Distribution of Lespedeza capitata

Plants Per Quadrat	Number of Quadrats				
	Observed	Calculated			
		Moment Solutions		Max. Likelihood Solutions	
		Thomas I	Neyman	Thomas II	Neyman*
0	7178	7279.2	7265.1	7178.0	7188.4
1	286	105.2	134.0	286.0	219.6
2	93	127.9	118.9	133.9	140.8
3	40	78.6	71.1	33.9	61.6
4	24	33.1	32.5	6.7	21.1
5	7	11.1	12.3	1.2	6.2
6	5	3.4	4.2	0.3	1.7
7	1	1.0	1.3		0.5
8	2	0.5	0.4		0.1
9	1		0.1		....
10	2		....		....
11	1		....		....
12 or over	0		....		....
Total.	7640	7640.0	7640.0	7640.0	7640.0
Plants per cluster. .	....	2.208	1.756	1.448	1.252
Clusters per quadrat	....	0.0484	0.0608	0.0624	0.0853
Plants per quadrat. .	....	0.1068	0.1068	0.0903	0.1068

*Shenton's method.

more regular than random and that the Lespedeza is somewhat clumped and point up the difference between the results from the two contagious series models. It is evident that these clusters are very much smaller than the major clumps which are apparent to the eye on a casual observation of the plants themselves. If these species had been randomly distributed the number of plants per cluster would have been one, corresponding to the Poisson distribution. The divergence of this parameter from unity shows that the plants are not so distributed, but the nature and type of nonrandomness is not the same as the expectation suggested by the general appearance of the charts.

It is probable that the minor clusters of the mathematical models are only related to very local clumping conditions. This conclusion is partly confirmed by an examination of the position plot for Solidago, where a large number of very small clumps are evident; the major clumps are distinctive and easily differentiated from these local aggregates (see also Cain and Evans, 1952).

MEASURES OF CLUMPING

A number of authors have considered the general problem of measures of clumping without any marked degree of success. Much of the recent literature on phytosociological characters has been summarized by Curtis and McIntosh (1950), who consider at length the meaning and meanings of the fundamental terms such as frequency (proportion of quadrats containing species to total quadrats), density (individuals per unit area), mean area (area per individual), and abundance (density divided by frequency). They did not make a detailed study of nonrandomness for their synthetic population, although they discovered that the measures of nonrandomness proposed by McGinnies (1934), Whitford (1949), Blackman (1942), and Fracker and Brischle (1944) were all dependent on quadrat size when the species were actually nonrandom.

It must be kept in mind that while disagreement of these measures with the expected values from the Poisson distribution certainly indicates some kind of nonrandomness of arrangement, agreement does not necessarily imply that the distribution of the plants is random since it is simple to devise non-Poisson distributions such that the measures are the same as those for the Poisson.

The values of $V/\bar{x}$ and K_M, K_F, and K_W in Table V provide these four measures of nonrandomness for the one-meter-quadrat data on the three old-field species under consideration. These measures (and the contagious series results) show that the three species have the following types of distribution:

Index	Most Random to Least Random				
Thomas	Solidago	$>$	Liatris	$>$	Lespedeza
Blackman*	Solidago	$>$	Liatris	$>$	Lespedeza
McGinnies	Solidago	$>$	Liatris	$>$	Lespedeza
Fracker	Solidago	$>$	Liatris	$\geqslant$	Lespedeza
Whitford	Solidago	$\geqslant$	Lespedeza	$>$	Liatris

* $V/\bar{x}$

The new measure of spacing introduced by Dice (1952) leads to the same conclusion as the measures from the contagious distributions.

These conclusions can be compared with the general appearance of the community as judged by eye from the plots:

Solidago $>$ Liatris $>$ Lespedeza

It is evident that for these species most of these measures arrive at the same conclusion as the visual estimation. The anomalous result from Whitford's index is not surprising in view of its sensitivity to quadrat size (Curtis and McIntosh, 1950). There is also no definite value for complete randomness. This is in sharp contrast to the other measures.

TABLE V

Parameters of the Sampled Plant Populations

	Solidago	Liatris	Lespedeza
Mean $\bar{x}$ or density D	0.06191	0.04188	0.10681
Variance V	0.07640	0.07285	0.29435
Frequency F (per cent)	5.380	3.102	6.047
Abundance $\underline{A}$	1.151	1.350	1.766
Expected density* $\underline{d}$ . .	0.05530	0.03151	0.06238
Measures of randomness†			
Variance/mean $V/\bar{x}$ . .	1.234	1.739	2.756
$K_M = D/d$	1.12	1.33	1.71
$K_F = (D - d)/d^2$	2.16	10.45	11.42
$K_W = 100D/F^2$	0.214	0.435	0.292

* Calculated from the frequency. See Fracker (1944, Table I).

† $V/\bar{x}$ Blackman (1942), etc. Random: $V/\bar{x} = 1$
K_M McGinnies (1934). Random: $K_M = 1$
K_F Fracker (1944). Random: $K_F = 0$
K_W Whitford (1949). Random: K_W is small (0 to 0.3)

SUMMARY

The contagious distributions of Neyman (1939) and Thomas (1949) were fitted to quadrat frequency data on Solidago rigida, Liatris aspera, and Lespedeza capitata in an old-field community. The goodness of fit was excellent, fair, and very bad, respectively. The size of the contagious distribution "clusters" of the mathematical model, 1 to 2 plants per cluster, had little relation to the evident major clumping obvious to the eye, but is more likely related to local clumping effects.

A number of different measures of aggregation or clumping which have been suggested in the literature are discussed. None of these indicates the actual size of the clumps. This seems to be a new problem demanding further research. The relative amount of major clumping of the three species noted by observation was compared with that shown by these different measures and was found to be the same with Lespedeza most clumped, Solidago least clumped, and Liatris in between.

An appendix presents and exemplifies a method for the calculation of the individual terms of the contagious series by means of direct calculation of polynomials instead of by the use of summations.

APPENDIX

Calculation of Terms of Contagious-distribution Series

It is possible that the ecologists who might have tried Neyman's or Thomas' series as a possible mathematical model for quadrat frequency data were deterred by the mathematical complexity of these series. This section shows that it is possible to evaluate the individual calculated terms of these series by the use of simple polynomial functions of variables which can be determined directly from the experimental data. Both series require as starting values the mean and variance of the observed distribution. These are obtained by the usual standard procedures which are described below.

Mean and Variance

Let f be the number of quadrats which contain X plants per quadrat (X = 0, 1, 2, ...), then the mean or average number of plants per quadrat is:

$$\mu_1' = \Sigma(fX)/\Sigma f$$

The summations Σ extend over all the values of X.

The corresponding variance or second moment of the distribution may be most accurately and usually also most easily obtained from the relation:

$$\mu_2 = \frac{\Sigma(fX^2)}{\Sigma f} - \left(\frac{\Sigma(fX)}{\Sigma f}\right)^2$$

As an example consider the data on the distribution of Solidago rigida in the old field (Table VI).

TABLE VI

Calculation of Mean and Variance

Number of Quadrats f	Plants Per Quadrat X	fX	fX²
7229	0	0	0
356	1	356	356
49	2	98	196
5	3	15	45
1	4	4	16
0	>4	0	0
Total 7640		473	613

The summations are:

$$\Sigma f = 7640 \qquad \Sigma fX = 473 \qquad \Sigma fX^2 = 613$$

The mean is:

$$\mu_1' = 473/7640 = 0.061911$$

The variance is:

$$\mu_2 = \frac{613}{7640} - \left(\frac{473}{7640}\right)^2 = 0.076403$$

Neyman's Type A Contagious Distribution

Beall (1939) has shown that calculated values of Neyman's (1939) "Type A Contagious Distribution" can be evaluated by a simple method involving, for each term, the summation of a linear function of the preceding terms of the distribution. This section presents a direct solution such that each calculated value of the series can be obtained from a polynomial in a new variable without the necessity of calculating the preceding terms. The relation was derived by direct substitution in Beall's formulas.

Let m_1 = Neyman's parameter, proportional to the number of "clusters" per quadrat

m_2 = Neyman's parameter, proportional to the number of plants per "cluster"

μ_1' = mean number of plants per quadrat

μ_2 = variance

n_i = calculated number of quadrats each of which contains i plants

n = total number of quadrats, the sum of the n_i

Neyman has shown that m_1 and m_2 may be estimated from the mean μ_1' and variance μ_2 of the observed values by the relations:

$$m_2 = (\mu_2 - \mu_1')/\mu_1'$$

$$m_1 = -\mu_1'/m_2 = (\mu_1')^2/(\mu_2 - \mu_1')$$

This is the solution obtained by equating the moments of the theoretical distribution and the moments of the observed distribution. The maximum likelihood solution has been given by Shenton (1949). This method is fairly complicated and requires the moment solution as part of the calculation. Shenton's paper includes a worked out example. Beall's relations for the calculated values are:

$$n_0 = ne^{-m_1(1-e^{-m_2})}$$

$$n_{i+1} = \frac{m_1 m_2 e^{-m_2}}{i+1} \sum_{t=0}^{t=i} \frac{m_2^{t}}{t!} n_{i-t}$$

The present relations use a new variable, U, which is defined as follows:

$$U \equiv m_1 e^{-m_2}$$

The value of n_1 is:

$$n_1 = m_2 n_0 U$$

The higher terms n_2, n_3, ... are obtained from expressions such as:

$$n_2 = n_1 \frac{m_2}{2!} (1 + U)$$

$$n_3 = n_1 \frac{m_2^{2}}{3!} (1 + 3U + U^2)$$

$$n_4 = n_1 \frac{m_2^{3}}{4!} (1 + 7U + 6U^2 + U^3)$$

$$n_5 = n_1 \frac{m_2^{4}}{5!} (1 + 15U + 25U^2 + 10U^3 + U^4)$$

$$n_6 = n_1 \frac{m_2^{5}}{6!} (1 + 31U + 90U^2 + 65U^3 + 15U^4 + U^5)$$

that is

$$n_i = n_1 \frac{m_2^{i-1}}{i!} (\text{polynomial in } U^{i-1})$$

The coefficients of the polynomials with the appropriate values of the factorials (i!) are shown in Table VII. The use of these values with a table of powers of m_2 and U permits the direct calculation of any n_i.

Thomas' Double Poisson Series

The double Poisson series of Thomas (1949) is also presented as a summation.

$$n_i = n \sum_{r=1}^{r=i} \frac{m^r e^{-m} (r\lambda)^{i-r} e^{-r\lambda}}{r! \quad (i-r)!}$$

where m = Thomas' parameter, the number of "clusters" per quadrat

λ = Thomas' parameter, one less than the number of plants per "cluster"

n_i = calculated number of quadrats each of which contains i plants

n = total number of quadrats, the sum of the n_i.

This equation may be written in the equivalent form which facilitates the direct calculation:

$$n_i = ne^{-m} \sum_{r=1}^{r=i} \frac{r^{i-r}}{r! \quad (i-r)!} \quad \lambda^{i-r} (me^{-\lambda})^r$$

since the first factor after the summation term is independent of m and λ which must be estimated from the observations. Thomas gives two methods of making this estimation. Method I involves equating the mean μ_1' and variance of the sample to the mean and variance of the distribution. (This is the "moment" solution similar to that used by Neyman for the other series and described above.)

$$\text{Let} \quad g = \mu_2/\mu_1'$$

$$\text{then} \quad 2\lambda = g - 3 + \sqrt{g^2 - 2g + 5}$$

$$\text{and} \quad m = \mu_1'/(1 + \lambda)$$

Method II, the maximum likelihood estimate, involves only the observed n_0 and n_1:

$$e^{-m} = n_0/n$$

$$e^{-\lambda} = n_1/n_0 m$$

Once m and λ are established the calculated values of n_i can be computed by means of simple polynomials in an analogous manner to the treatment for Neyman's series above. A new variable $V \equiv me^{-\lambda}/\lambda$ is introduced. Direct substitution leads to the following relations:

$$n_0 = ne^{-m}$$

$$\begin{aligned} n_1 &= nme^{-m}e^{-\lambda} \\ &= n_0 \quad \lambda V \end{aligned}$$

$$n_2 = n_1 \frac{\lambda}{2!} (2 + V)$$

$$n_3 = n_1 \frac{\lambda^2}{3!} (3 + 6V + V^2)$$

$$n_4 = n_1 \frac{\lambda^3}{4!} (4 + 24V + 12V^2 + V^3)$$

$$n_5 = n_1 \frac{\lambda^4}{5!} (5 + 80V + 90V^2 + 20V^3 + V^4)$$

$$n_6 = n_1 \frac{\lambda^5}{6!} (6 + 240V + 540V^2 + 240V^3 + 30V^4 + V^5)$$

that is:

$$n_i = \frac{n_1 \lambda^{i-1}}{i!} \text{ (polynomial in } V^{i-1}\text{)}$$

The coefficients of the polynomials with the appropriate values of the factorials are given in Table VIII. The use of these values with a table of powers of λ and V permits the direct calculation of any n_i, without the necessity of considering the previous terms.

Example: As an example of the use of these polynomials the calculation of the terms in the Neyman's Type A series for the Solidago rigida data is shown below.

$$\mu'_1 = 0.061911$$

$$\mu_2 = 0.076403$$

$$m_2 = (0.076403 - 0.061911)/0.061911$$
$$= 0.23408$$

$$m_1 = 0.061911/0.23408 = 0.26449$$

$$e^{-m_2} = 0.79130$$

$$m_1(1 - e^{-m_2}) = (0.26449)\ (0.20870) = 0.055199$$

$$e^{-m_1(1 - e^{-m_2})} = 0.94630$$

$$n_0 = (7640)\ (0.96430) = 7229.73$$

$$U = (0.26449)\ (0.79130) = 0.20929$$

$$n_1 = (0.23408)\ (7229.73)\ (0.20929) = 354.19$$

The remaining terms are evaluated using the polynomials. First a table of powers of U and m_2 is prepared:

U	m_2	Factorial
$U = 0.20929$	$m_2 = 0.23408$	2
$U^2 = 0.043802$	$m_2^2 = 0.054793$	6
$U^3 = 0.0091674$	$m_2^3 = 0.012826$	24
$U^4 = 0.0019186$	$m_2^4 = 0.0030023$	720
etc.	etc.	etc.

It is convenient to jot down the appropriate factorial opposite the proper power of m_2 (as has been done above) since it will not only be close to the place where it will be used but also serves to locate the proper polynomial in Table VIII.

The calculation of n_5 serves as an example of the use of the polynomials:

$$n_5 = (354.19)\,(0.0030023/120)\,(1 + (15)\,(0.20929) + (25)\,(0.043802) + (10)\,(0.0091674) + 0.0019186) = 0.047$$

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TABLE VII

Coefficients of Powers of U in Polynomials for Neyman's Type A Contagious Distribution

	I	U	U^2	U^3	U^4	U^5	U^6	U^7	U^8	U^9	U^{10}	U^{11}	U^{12}	U^{13}	U^{14}	Factorial
n_2	1	1														2! = 2
n_3	1	3	1													3! = 6
n_4	1	7	6	1												4! = 24
n_5	1	15	25	10	1											5! = 120
n_6	1	31	90	65	15	1										6! = 720
n_7	1	63	301	350	140	21	1									7! = 5 040
n_8	1	127	966	1 701	1 050	266	28	1								8! = 4.0320 x 10^4
n_9	1	255	3 025	7 770	6 951	2 646	462	36	1							9! = 3.6288 x 10^5
n_{10}	1	511	9 330	34 105	42 525	22 827	5 880	750	45	1						10! = 3.6288 x 10^6
n_{11}	1	1 023	28 501	145 750	246 730	179 487	63 987	11 880	1 155	55	1					11! = 3.9917 x 10^7
n_{12}	1	2 047	86 526	611 501	1 379 400	1 323 652	627 396	159 027	22 275	1 705	66	1				12! = 4.7900 x 10^8
n_{13}	1	4 095	261 625	2 532 530	7 508 501	9 321 312	5 715 424	1 899 612	359 502	39 325	2 431	78	1			13! = 6.2270 x 10^9
n_{14}	1	8 191	788 970	10 391 745	40 075 035	63 436 373	49 329 280	20 912 320	5 135 130	752 752	66 066	3 367	91	1		14! = 8.7178 x 10^{10}
n_{15}	1	16 383	2 375 101	42 355 950	210 766 920	420 693 273	408 741 333	216 627 840	67 128 490	12 662 650	1 479 478	106 470	4 550	105	1	15! = 1.3077 x 10^{12}

TABLE VIII

Coefficients of Powers of V in Polynomials for Thomas' Double Poisson Distribution

	I	V	V^2	V^3	V^4	V^5	V^6	V^7	V^8	V^9	V^{10}	V^{11}	V^{12}	V^{13}	V^{14}	Factorial
n_2	2	1														2! = 2
n_3	3	6	1													3! = 6
n_4	4	24	12	1												4! = 24
n_5	5	80	90	20	1											5! = 120
n_6	6	240	540	240	30	1										6! = 720
n_7	7	672	2 835	2 240	525	42	1									7! = 5 040
n_8	8	1 792	13 608	17 920	7 000	1 008	56	1								8! = 4.0320 x 10^4
n_9	9	4 608	61 236	129 024	78 750	18 144	1 764	72	1							9! = 3.6288 x 10^5
n_{10}	10	11 520	262 440	860 160	787 500	272 160	41 160	2 880	90	1						10! = 3.6288 x 10^6
n_{11}	11	28 160	1 082 565	5 406 720	7 218 750	3 592 512	792 330	84 480	4 455	110	1					11! = 3.9917 x 10^7
n_{12}	12	67 584	4 330 260	32 440 320	61 875 000	43 110 144	13 311 144	2 027 520	160 380	6 600	132	1				12! = 4.7900 x 10^8
n_{13}	13	159 744	16 888 014	187 432 960	502 734 375	480 370 176	201 885 684	42 172 416	4 691 115	286 000	9 438	156	1			13! = 6.2270 x 10^9
n_{14}	14	372 736	64 481 508	1 049 624 576	3 910 156 250	5 043 886 848	2 826 399 576	787 218 432	118 216 098	10 010 000	484 484	13 104	182	1		14! = 8.7178 x 10^{10}
n_{15}	15	860 160	241 805 655	5 725 224 960	29 326 171 875	50 438 367 980	37 096 494 435	13 495 173 120	2 659 862 205	300 300 000	19 984 965	786 240	17 745	210	1	15! = 1.3077 x 10^{12}